SADLER UNIT3 CHAPTER 7

EXERCISE 7A

Q1. $\underset{\sim}{r}(t) = \begin{pmatrix} 2t^3 \\ 3t+1 \end{pmatrix}$

a) $\underset{\sim}{r}(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} m$

b) $\underset{\sim}{v}(t) = \begin{pmatrix} 6t^2 \\ 3 \end{pmatrix}$

$\underset{\sim}{v}(3) = \begin{pmatrix} 54 \\ 3 \end{pmatrix} m/s.$

c) $|\underset{\sim}{v}(3)| = \left|\begin{pmatrix} 54 \\ 3 \end{pmatrix}\right|$

$= \sqrt{2925}$

≈ 54.08 m/s.

d) $\underset{\sim}{a}(t) = \begin{pmatrix} 12t \\ 0 \end{pmatrix}$

$\underset{\sim}{a}(3) = \begin{pmatrix} 36 \\ 0 \end{pmatrix} m/s^2$

Q2. $\underset{\sim}{a}(t) = \begin{pmatrix} 6t \\ 0 \end{pmatrix}$

$\underset{\sim}{r}(0) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$\underset{\sim}{v}(0) = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$

$\underset{\sim}{v}(t) = \begin{pmatrix} 3t^2 \\ 0 \end{pmatrix} + \underset{\sim}{c}_1$

$\underset{\sim}{c}_1 = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$

$\therefore \underset{\sim}{v}(t) = \begin{pmatrix} 3t^2 - 4 \\ 6 \end{pmatrix}$

$\underset{\sim}{r}(t) = \begin{pmatrix} t^3 - 4t \\ 6t \end{pmatrix} + \underset{\sim}{c}_2$

$\underset{\sim}{c}_2 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$\therefore \underset{\sim}{r}(t) = \begin{pmatrix} t^3 - 4t + 2 \\ 6t - 1 \end{pmatrix}$

a) $\underset{\sim}{v}(2) = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$

$|\underset{\sim}{v}(2)| = \left|\begin{pmatrix} 8 \\ 6 \end{pmatrix}\right|$

$= \sqrt{64+36}$

$= 10$ m/s.

b) $\underset{\sim}{r}(2) = \begin{pmatrix} 8-8+2 \\ 11 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 11 \end{pmatrix}$

$\left|\begin{pmatrix} 2 \\ 11 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \end{pmatrix}\right| = \left|\begin{pmatrix} 0 \\ 12 \end{pmatrix}\right|$

$= 12$ m

Q3. $\underset{\sim}{r}(t) = \begin{pmatrix} 2t \\ t-1 \end{pmatrix}$

a) $\dfrac{d\underset{\sim}{r}}{dt} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\left|\dfrac{d\underset{\sim}{r}}{dt}\right| = \sqrt{4+1}$

$= \sqrt{5}$

b) $|\underset{\sim}{r}| = \sqrt{4t^2 + (t-1)^2}$

$= \sqrt{4t^2 + t^2 - 2t + 1}$

$= \sqrt{5t^2 - 2t + 1}$

$\dfrac{d|\underset{\sim}{r}|}{dt} = \frac{1}{2}(5t^2 - 2t + 1)^{-\frac{1}{2}}(10t - 2)$

$= \dfrac{5t - 1}{\sqrt{5t^2 - 2t + 1}}$

Q4. $\underset{\sim}{v}(t) = \begin{pmatrix} \frac{-1}{(t+1)^2} \\ 2 \end{pmatrix}$

a) $\underset{\sim}{v}(1) = \begin{pmatrix} -\frac{1}{4} \\ 2 \end{pmatrix}$

b) $\underset{\sim}{a}(t) = \begin{pmatrix} 2(t+1)^{-3} \\ 0 \end{pmatrix}$

$\underset{\sim}{a}(1) = \begin{pmatrix} \frac{2}{8} \\ 0 \end{pmatrix}$

$= \begin{pmatrix} \frac{1}{4} \\ 0 \end{pmatrix}$

c) $\underset{\sim}{r}(t) = \begin{pmatrix} \frac{1}{(t+1)} \\ 2t \end{pmatrix} + \underset{\sim}{c}$

$\underset{\sim}{r}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \underset{\sim}{c}$

$\begin{pmatrix} 3 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \underset{\sim}{c}$

$\underset{\sim}{c} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$\therefore \underset{\sim}{r}(t) = \begin{pmatrix} \frac{1}{t+1} + 2 \\ 2t + 3 \end{pmatrix}$

$\underset{\sim}{r}(1) = \begin{pmatrix} \frac{5}{2} \\ 5 \end{pmatrix}$

Q5

$\underset{\sim}{r}(t) = \begin{pmatrix} t^2 - 5t + 1 \\ 1 - 14t + t^2 \end{pmatrix}$

a) $\underset{\sim}{v}(t) = \begin{pmatrix} 2t - 5 \\ 2t - 14 \end{pmatrix}$

$\begin{pmatrix} k \\ 0 \end{pmatrix} = \begin{pmatrix} 2t - 5 \\ 2t - 14 \end{pmatrix}$

$\therefore 2t - 14 = 0$

$t = 7$ secs.

b) $\begin{pmatrix} 0 \\ k \end{pmatrix} = \begin{pmatrix} 2t - 5 \\ 2t - 14 \end{pmatrix}$

$\therefore 2t - 5 = 0$

$t = 2.5$ secs.

Q6. $\underset{\sim}{v}(t) = \begin{pmatrix} 2 \\ e^{0.1t} \end{pmatrix}$

a) $\underset{\sim}{v}(10) = \begin{pmatrix} 2 \\ e \end{pmatrix}$ m/s.

b) $\underset{\sim}{a}(t) = \begin{pmatrix} 0 \\ 0.1e^{0.1t} \end{pmatrix}$

$\underset{\sim}{a}(10) = \begin{pmatrix} 0 \\ 0.1e \end{pmatrix} m/s^2$

c)

$$\underset{\sim}{r}(t) = \begin{pmatrix} 2t \\ 10e^{0.1t} \end{pmatrix} + \underset{\sim}{c}$$

$$\begin{pmatrix} 0 \\ 10 \end{pmatrix} = \begin{pmatrix} 0 \\ 10 \end{pmatrix} + \underset{\sim}{c}$$

$$\underset{\sim}{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\therefore \underset{\sim}{r}(t) = \begin{pmatrix} 2t \\ 10e^{0.1t} \end{pmatrix}$$

$$\underset{\sim}{r}(10) = \begin{pmatrix} 20 \\ 10e \end{pmatrix} \text{m}$$

Q7.

$$\underset{\sim}{r}(t) = \begin{pmatrix} 8t-12 \\ t^2 \end{pmatrix}$$

a) $\underset{\sim}{r}(3) = \begin{pmatrix} 12 \\ 9 \end{pmatrix}$ m.

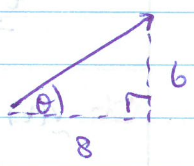
$$|\underset{\sim}{r}(3)| = \sqrt{144+81} = \sqrt{225} = 15\text{m}$$

b) $\underset{\sim}{v}(t) = \begin{pmatrix} 8 \\ 2t \end{pmatrix}$

$$\underset{\sim}{v}(3) = \begin{pmatrix} 8 \\ 6 \end{pmatrix} \text{m/s}$$

c) $|\underset{\sim}{v}(3)| = \sqrt{64+36} = 10$ m/s.

d)



$$\tan\theta = \frac{6}{8}$$

$$\theta = \tan^{-1}\left(\frac{6}{8}\right) = 36.87^\circ \approx 37^\circ$$

Q8

$$\underset{\sim}{r}(t) = \begin{pmatrix} t^3 \\ 2t^2-1 \end{pmatrix}$$

a)

$$\underset{\sim}{v}(t) = \begin{pmatrix} 3t^2 \\ 4t \end{pmatrix}$$

$$\underset{\sim}{v}(2) = \begin{pmatrix} 12 \\ 8 \end{pmatrix}$$

$$|\underset{\sim}{v}(2)| = \sqrt{144+64} = \sqrt{208} = 14.42 \text{ m/s}.$$

b)

$$\underset{\sim}{a}(t) = \begin{pmatrix} 6t \\ 4 \end{pmatrix}$$

$$\underset{\sim}{a}(3) = \begin{pmatrix} 18 \\ 4 \end{pmatrix} \text{m/s}^2$$

c)

$$\underset{\sim}{v} \cdot \underset{\sim}{a} = \begin{pmatrix} 3t^2 \\ 4t \end{pmatrix}\begin{pmatrix} 6t \\ 4 \end{pmatrix} = 18t^3 + 16t$$

$$\underset{\sim}{v} \cdot \underset{\sim}{a}(2) = 18(8) + 16(2) = 144 + 32 = 176.$$

d)

$$\cos\theta = \frac{\underset{\sim}{v} \cdot \underset{\sim}{a}}{|\underset{\sim}{v}||\underset{\sim}{a}|} = \frac{176}{\sqrt{208}\sqrt{160}}$$

$$\theta = \cos^{-1}\left(\frac{176}{\sqrt{33280}}\right) = 15.26^\circ \approx 15.3^\circ$$

Q9. $\underset{\sim}{v}(t) = \begin{pmatrix} 2t \\ 3t^2-1 \\ -3 \end{pmatrix}$

a) $\underset{\sim}{v}(0) = \begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix}$

$$|\underset{\sim}{v}(0)| = \sqrt{1+9} = \sqrt{10} \text{ m/s}.$$

b) $\underset{\sim}{v}(2) = \begin{pmatrix} 4 \\ 11 \\ -3 \end{pmatrix}$

$$|\underset{\sim}{v}(2)| = \sqrt{16+121+9} = \sqrt{146} \text{ m/s}.$$

c) $\underset{\sim}{a}(t) = \begin{pmatrix} 2 \\ 6t \\ 0 \end{pmatrix}$

$$\underset{\sim}{a}(2) = \begin{pmatrix} 2 \\ 12 \\ 0 \end{pmatrix} \text{m/s}^2$$

d) $\underset{\sim}{r}(t) = \begin{pmatrix} t^2 \\ t^3-t \\ -3t \end{pmatrix} + \underset{\sim}{c}$

$$\begin{pmatrix} -4 \\ 10 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ -6 \end{pmatrix} + \underset{\sim}{c}$$

$$\underset{\sim}{c} = \begin{pmatrix} -8 \\ 4 \\ 6 \end{pmatrix}$$

$$\therefore \underset{\sim}{r}(t) = \begin{pmatrix} t^2-8 \\ t^3-t+4 \\ -3t+6 \end{pmatrix}$$

$$\underset{\sim}{r}(5) = \begin{pmatrix} 17 \\ 124 \\ -9 \end{pmatrix} \text{m}$$

Q10.

$\underset{\sim}{r}(t) = \begin{pmatrix} t^2-6t-16 \\ t^2 \end{pmatrix}$

a) $\begin{pmatrix} 0 \\ k \end{pmatrix} = \begin{pmatrix} t^2-6t-16 \\ t^2 \end{pmatrix}$

$t^2-6t-16=0$

$(t-8)(t+2)=0$

$\underline{\underline{t=8}}$ and $t=-2$ (reject)

b) $\underset{\sim}{v}(t) = \begin{pmatrix} 2t-6 \\ 2t \end{pmatrix}$

$\begin{pmatrix} 0 \\ k \end{pmatrix} = \begin{pmatrix} 2t-6 \\ 2t \end{pmatrix}$

$2t-6=0$

$\underline{\underline{t=3}}$

c) $\underset{\sim}{a}(t) = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

$\underset{\sim}{v} \cdot \underset{\sim}{a} = 0$

$\begin{pmatrix} 2t-6 \\ 2t \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 2 \end{pmatrix} = 0$

$4t-12+4t=0$

$8t-12=0$

$t = \frac{12}{8}$

$\underline{\underline{t=1.5}}$

Q11.

$\underset{\sim}{r}(t) = \begin{pmatrix} 3 \\ 2t \\ t^2-4t+10 \end{pmatrix}$

$\frac{d}{dt}(t^2-4t+10)$

$2t-4=0$

$\underline{\underline{t=2}}$

$\underset{\sim}{r}(2) = \underline{\underline{\begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix}}}$ m

$\underset{\sim}{v}(t) = \begin{pmatrix} 0 \\ 2 \\ 2t-4 \end{pmatrix}$

$\underset{\sim}{v}(2) = \underline{\underline{\begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}}}$ m/s.

$\underset{\sim}{a}(t) = \underline{\underline{\begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}}}$ m/s^2

Q12. $\underset{\sim}{a}(t) = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$

$\underset{\sim}{r}(0) = \begin{pmatrix} 1 \\ 20 \end{pmatrix}$

$\underset{\sim}{v}(0) = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$

a) $\underset{\sim}{v}(t) = \begin{pmatrix} 0 \\ 2t \end{pmatrix} + \underset{\sim}{c}_1$

$\therefore \underset{\sim}{v}(t) = \underline{\underline{\begin{pmatrix} 2 \\ 2t-8 \end{pmatrix}}}$

b) $\underset{\sim}{r}(t) = \begin{pmatrix} 2t \\ t^2-8t \end{pmatrix} + \underset{\sim}{c}_2$

$\begin{pmatrix} 1 \\ 20 \end{pmatrix} = \underset{\sim}{c}_2$

$\underset{\sim}{r}(t) = \underline{\underline{\begin{pmatrix} 2t+1 \\ t^2-8t+20 \end{pmatrix}}}$

c) $\underset{\sim}{r}(3) = \begin{pmatrix} 7 \\ 5 \end{pmatrix}$

$|\underset{\sim}{r}(3)| = \sqrt{49+25}$

$= \underline{\underline{\sqrt{74}}}$ m

d) $\underset{\sim}{v}(2) = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$

$|\underset{\sim}{v}(2)| = \sqrt{4+16}$

$= \sqrt{20}$

$= \underline{\underline{2\sqrt{5}\text{ m/s}}}$

e) $\frac{d}{dt}(t^2-8t+20)$

$= 2t - 8$

$0 = 2t-8$

$\underline{\underline{t=4}}$

$t^2-8t+20 \,\Big|_{t=4}$

$= 16-32+20$

$= \underline{\underline{4\text{ m}}}$

f) $x = 2t+1$

$y = t^2-8t+20$

$\frac{x-1}{2} = t$

$\therefore y = \left(\frac{x-1}{2}\right)^2 - 8\left(\frac{x-1}{2}\right)+20$

$y = \frac{(x-1)^2}{4} - 4(x-1)+20$

$y = \frac{x^2}{4} - \frac{x}{2} + \frac{1}{4} - 4x + 4 + 20$

$= \frac{x^2}{4} - \frac{9x}{2} + \frac{97}{4}$

Q13. $\underset{\sim}{a}(t) = \begin{pmatrix} \cos t \\ 2 \end{pmatrix}$

$\underset{\sim}{v}(0) = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$

$\underset{\sim}{r}(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\underset{\sim}{v}(t) = \begin{pmatrix} \sin t \\ 2t \end{pmatrix} + \underset{\sim}{c}_1$

$\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + c_1$

$\underset{\sim}{v}(t) = \begin{pmatrix} \sin t \\ 2t+1 \end{pmatrix}$

$\underset{\sim}{r}(t) = \begin{pmatrix} -\cos t \\ t^2+t \end{pmatrix} + \underset{\sim}{c}_2$

$$\begin{pmatrix}4\\-6\end{pmatrix} = \begin{pmatrix}-1\\0\end{pmatrix} + \underset{\sim}{c}_2$$

$$\begin{pmatrix}5\\-6\end{pmatrix} = \underset{\sim}{c}_2$$

$$\therefore \underset{\sim}{r}(t) = \begin{pmatrix}5-\cos t\\t^2+t-6\end{pmatrix}$$

a) $\begin{pmatrix}5-\cos t\\t^2+t-6\end{pmatrix} = \begin{pmatrix}k\\0\end{pmatrix}$

$t^2+t-6=0$

$(t-2)(t+3)=0$

$t=2$ or $t=-3$ (reject)

b) $\begin{pmatrix}5-\cos t\\t^2+t-6\end{pmatrix} = \begin{pmatrix}0\\k\end{pmatrix}$

$5-\cos t=0$

$\Rightarrow$ never touches y axis

Q14.

$$\underset{\sim}{a}(t) = \begin{pmatrix}-4\sin 2t\\2\\e^t\end{pmatrix}$$

$$\underset{\sim}{r}(0) = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$\underset{\sim}{v}(0) = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$\underset{\sim}{v}(t) = \begin{pmatrix}2\cos 2t\\2t\\e^t\end{pmatrix} + \underset{\sim}{c}_1$$

$$\begin{pmatrix}0\\0\\0\end{pmatrix} = \begin{pmatrix}2\\0\\1\end{pmatrix} + \underset{\sim}{c}_1$$

$$\underset{\sim}{v}(t) = \begin{pmatrix}2\cos 2t-2\\2t\\e^t-1\end{pmatrix}$$

$$\underset{\sim}{r}(t) = \begin{pmatrix}\sin 2t-2t\\t^2\\e^t-t\end{pmatrix} + \underset{\sim}{c}_2$$

$$\begin{pmatrix}0\\0\\0\end{pmatrix} = \begin{pmatrix}0\\0\\1\end{pmatrix} + \underset{\sim}{c}_2$$

$$\underset{\sim}{r}(t) = \begin{pmatrix}\sin 2t-2t\\t^2\\e^t-t-1\end{pmatrix}$$

$$\underset{\sim}{r}(\pi) = \begin{pmatrix}-2\pi\\\pi^2\\e^\pi-\pi-1\end{pmatrix}\text{m}$$

Q15

$$\underset{\sim}{r} = \begin{pmatrix}2\sin 3t\\2\cos 3t\end{pmatrix}$$

a) $\begin{pmatrix}k\\0\end{pmatrix} = \begin{pmatrix}2\sin 3t\\2\cos 3t\end{pmatrix}$

$2\cos 3t=0$

$\cos 3t=0$

$3t=\frac{\pi}{2}$

$t=\frac{\pi}{6}$

b) $\underset{\sim}{v}(t) = \begin{pmatrix}6\cos 3t\\-6\sin 3t\end{pmatrix}$

$\underset{\sim}{a}(t) = \begin{pmatrix}-18\sin 3t\\-18\cos 3t\end{pmatrix}$

$= -9\begin{pmatrix}2\sin 3t\\2\cos 3t\end{pmatrix}$

c) $\begin{pmatrix}6\cos 3t\\-6\sin 3t\end{pmatrix}\cdot\begin{pmatrix}-18\sin 3t\\-18\cos 3t\end{pmatrix}$

$= -108\cos 3t\sin 3t + 108\cos 3t\sin 3t$

$= 0$

$\therefore \underset{\sim}{v}$ always $\perp$ to $\underset{\sim}{a}$.

Q16. $\underset{\sim}{r}(0) = \begin{pmatrix}2\\8\end{pmatrix}$

$\underset{\sim}{v}(0) = \begin{pmatrix}-4\\0\end{pmatrix}$

$$\underset{\sim}{a}(t) = \begin{pmatrix}2\sin(0.5t)\\-2\cos(0.5t)\end{pmatrix}$$

$$\underset{\sim}{v}(t) = \begin{pmatrix}-4\cos(0.5t)\\-4\sin(0.5t)\end{pmatrix} + \underset{\sim}{c}_1$$

$$\begin{pmatrix}-4\\0\end{pmatrix} = \begin{pmatrix}-4\\0\end{pmatrix} + \underset{\sim}{c}_1$$

$$\underset{\sim}{c}_1 = \begin{pmatrix}0\\0\end{pmatrix}$$

$$\underset{\sim}{v}(t) = \begin{pmatrix}-4\cos(0.5t)\\-4\sin(0.5t)\end{pmatrix}$$

$$\underset{\sim}{r}(t) = \begin{pmatrix}-8\sin(0.5t)\\8\cos(0.5t)\end{pmatrix} + \underset{\sim}{c}_2$$

$$\begin{pmatrix}2\\8\end{pmatrix} = \begin{pmatrix}0\\8\end{pmatrix} + \underset{\sim}{c}_2$$

$$\underset{\sim}{c}_2 = \begin{pmatrix}2\\0\end{pmatrix}$$

$$\therefore \underset{\sim}{r}(t) = \begin{pmatrix}2-8\sin 0.5t\\8\cos 0.5t\end{pmatrix}$$

$$\therefore \underset{\sim}{r}\left(\tfrac{\pi}{3}\right) = \begin{pmatrix}2-8\sin\frac{\pi}{6}\\8\cos\frac{\pi}{6}\end{pmatrix}$$

$$= \begin{pmatrix}2-4\\4\sqrt{3}\end{pmatrix}$$

$$= \begin{pmatrix}-2\\4\sqrt{3}\end{pmatrix}$$

$$\left|\begin{pmatrix}-2\\4\sqrt{3}\end{pmatrix} - \begin{pmatrix}2\\0\end{pmatrix}\right|$$

$$= \left|\begin{pmatrix}-4\\4\sqrt{3}\end{pmatrix}\right|$$

$$= \sqrt{16+48}$$

$$= 8\text{m}$$

EXERCISE 7B

Q1

Let $\underset{\sim}{a}(t) = \begin{pmatrix} a \\ 0 \end{pmatrix}$

$\underset{\sim}{v}(t) = \begin{pmatrix} at \\ 0 \end{pmatrix} + \underset{\sim}{c}_1$

$\begin{pmatrix} u \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \underset{\sim}{c}_1$

$\therefore \underset{\sim}{v}(t) = \begin{pmatrix} at + u \\ 0 \end{pmatrix}$

$\underset{\sim}{r}(t) = \begin{pmatrix} \frac{at^2}{2} + ut \\ 0 \end{pmatrix} + \underset{\sim}{c}_2$

$\underset{\sim}{r}(t) = \begin{pmatrix} \frac{at^2}{2} + ut \\ 0 \end{pmatrix}$

Q2

$\underset{\sim}{a}(t) = \begin{pmatrix} 0 \\ -9.8 \end{pmatrix}$

$\underset{\sim}{v}(t) = \begin{pmatrix} 0 \\ -9.8t \end{pmatrix} + \underset{\sim}{c}_1$

$= \begin{pmatrix} 14 \\ -9.8t + 35 \end{pmatrix}$

$\underset{\sim}{r}(t) = \begin{pmatrix} 14t \\ -4.9t^2 + 35t \end{pmatrix}$

$\underset{\sim}{r}(5) = \begin{pmatrix} 70 \\ 52.5 \end{pmatrix}$

$|\underset{\sim}{r}(5)| = \sqrt{70^2 + 52.5^2}$

$= 87.5 \text{ m}$

$x = 14t \Rightarrow t = \frac{x}{14}$

$y = -4.9t^2 + 35t$

$= -4.9\left(\frac{x}{14}\right)^2 + 35\left(\frac{x}{14}\right)$

$= -\frac{4.9x^2}{196} + \frac{5x}{2}$

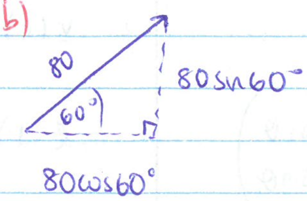
$= -\frac{x^2}{40} + \frac{5x}{2}$

Q3.

a) $\underset{\sim}{a}(t) = \begin{pmatrix} 0 \\ -10 \end{pmatrix} \text{ m/s}^2$

$= -10\underset{\sim}{j} \text{ m/s}^2$

b)



$\underset{\sim}{v}(0) = \begin{pmatrix} 80\left(\frac{1}{2}\right) \\ 80\left(\frac{\sqrt{3}}{2}\right) \end{pmatrix}$

$= \begin{pmatrix} 40 \\ 40\sqrt{3} \end{pmatrix} \text{ m/s.}$

$\therefore 40\underset{\sim}{i} + 40\sqrt{3}\underset{\sim}{j} \text{ m/s}$

c) $\underset{\sim}{v}(t) = \begin{pmatrix} 0 \\ -10t \end{pmatrix} + \underset{\sim}{c}$

$= \begin{pmatrix} 40 \\ -10t + 40\sqrt{3} \end{pmatrix}$

$\underset{\sim}{r}(t) = \begin{pmatrix} 40t \\ -5t^2 + 40\sqrt{3}t \end{pmatrix} + \underset{\sim}{c}_2$

$= 40t\underset{\sim}{i} + (40\sqrt{3}t - 5t^2)\underset{\sim}{j}$

m.

d) $-5t^2 + 40\sqrt{3}t = 0$

$-5t(t - 8\sqrt{3}) = 0$

$t = 0$ (ignore) or $t = 8\sqrt{3}$

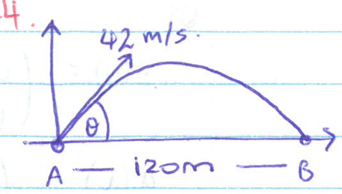
$= 13.86$ secs

e) $\underset{\sim}{r}(13.86) = \begin{pmatrix} 40(8\sqrt{3}) \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 320\sqrt{3} \\ 0 \end{pmatrix}$

$|\underset{\sim}{r}(13.86)| = 320\sqrt{3} \text{ m.}$

Q4.



$\underset{\sim}{a}(t) = \begin{pmatrix} 0 \\ -9.8 \end{pmatrix}$

a) $\underset{\sim}{v}(t) = \begin{pmatrix} 0 \\ -9.8t \end{pmatrix} + \underset{\sim}{c}_1$

$\underset{\sim}{c}_1 = \begin{pmatrix} 42\cos\theta \\ 42\sin\theta \end{pmatrix}$

$\underset{\sim}{v}(t) = \begin{pmatrix} 42\cos\theta \\ -9.8t + 42\sin\theta \end{pmatrix}$

$\underset{\sim}{r}(t) = \begin{pmatrix} (42\cos\theta)t \\ -4.9t^2 + 42\sin\theta\, t \end{pmatrix}$

b)

$120 = 42t\cos\theta$ (1)

$-4.9t^2 + 42\sin\theta\, t = 0$ (2)

$t = \frac{120}{42\cos\theta}$

$t(42\sin\theta - 4.9t) = 0$

$t = 0$ or $t = \frac{42\sin\theta}{4.9}$

$\frac{120}{42\cos\theta} = \frac{42\sin\theta}{4.9}$

$\frac{588}{42^2} = \sin\theta\cos\theta$

$\frac{1}{3} = \sin\theta\cos\theta$

$\frac{1}{3} = \frac{1}{2}\sin 2\theta$

$\sin 2\theta = \frac{2}{3}$

$2\theta = 41.81°$ and $138.19°$

$\theta = 20.91°$ and $69.10°$

Q5

$\underset{\sim}{v}(0) = \begin{pmatrix} u\cos\theta \\ u\sin\theta \end{pmatrix}$

$\underset{\sim}{a}(t) = \begin{pmatrix} 0 \\ -g \end{pmatrix}$

a) $u\cos\theta\,\underset{\sim}{i} + u\sin\theta\,\underset{\sim}{j}$

b) $\underset{\sim}{v}(t) = \begin{pmatrix} 0 \\ -gt \end{pmatrix} + \begin{pmatrix} u\cos\theta \\ u\sin\theta \end{pmatrix}$

$= \begin{pmatrix} u\cos\theta \\ u\sin\theta - gt \end{pmatrix}$

$\underset{\sim}{r}(t) = \begin{pmatrix} ut\cos\theta \\ ut\sin\theta - \frac{gt^2}{2} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} ut\cos\theta \\ ut\sin\theta - \frac{gt^2}{2} \end{pmatrix}$

c) $ut\sin\theta - \frac{gt^2}{2} = 0$

$t\left(u\sin\theta - \frac{gt}{2}\right) = 0$

$\underline{\underline{t=0}}$ and $\frac{gt}{2} = u\sin\theta$

$t = \underline{\underline{\frac{2u\sin\theta}{g}}}$

d) $= u\left(\frac{2u\sin\theta}{g}\right)\cos\theta$

$= \frac{2u^2\sin\theta\cos\theta}{g}$

$= \underline{\underline{\frac{u^2}{g}\sin 2\theta}}$ m

e) maximum when

$\sin 2\theta = 1$

$2\theta = \frac{\pi}{2}$

$\underline{\underline{\theta = \frac{\pi}{4}}}$

Q6.

$\underset{\sim}{r}(0) = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

$\underset{\sim}{r}(t) = \begin{pmatrix} 2\cos(0.5t) \\ 2\sin(0.5t) \end{pmatrix}$

a) $\underset{\sim}{v}(t) = \begin{pmatrix} -\sin(0.5t) \\ \cos(0.5t) \end{pmatrix}$ m/s

$\underset{\sim}{a}(t) = \begin{pmatrix} -0.5\cos(0.5t) \\ -0.5\sin(0.5t) \end{pmatrix}$ m/s^2

b) $|\underset{\sim}{v}(t)| = \sqrt{\sin^2(0.5t) + \cos^2(0.5t)}$

$= \sqrt{1}$

$= \underline{\underline{1}}$ m/s.

c) $\begin{pmatrix} -\sin 0.5t \\ \cos(0.5t) \end{pmatrix} \cdot \begin{pmatrix} -0.5\cos(0.5t) \\ -0.5\sin(0.5t) \end{pmatrix}$

$= 0.5\sin(0.5t)\cos(0.5t) - 0.5\sin(0.5t)\cos(0.5t)$

$= 0$.

∴ velocity and acceleration vectors always perpendicular to each other.

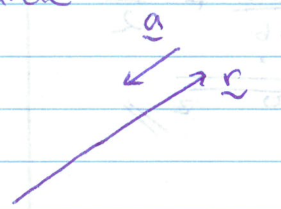
d) $\underset{\sim}{a}(t) = \begin{pmatrix} -0.5\cos(0.5t) \\ -0.5\sin(0.5t) \end{pmatrix}$

$= -\frac{1}{4}\begin{pmatrix} 2\cos 0.5t \\ 2\sin 0.5t \end{pmatrix}$

$= -\frac{1}{4}\underset{\sim}{r}(t)$.

$\therefore \underline{\underline{k = \frac{1}{4}}}$

e) Given that $\underset{\sim}{a}$ is always negative of the displacement, the acceleration vector is antiparallel to displacement ∴ acceleration is towards centre of circle



Q7. $\underset{\sim}{r}(0) = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$

$$\underset{\sim}{v}(t) = \begin{pmatrix} -\frac{5\pi}{2}\cos\left(\frac{\pi}{2}t\right) \\ -\frac{5\pi}{2}\sin\left(\frac{\pi}{2}t\right) \end{pmatrix}$$

a) $$\underset{\sim}{r}(t) = \begin{pmatrix} -5\sin\left(\frac{\pi}{2}t\right) \\ 5\cos\left(\frac{\pi}{2}t\right) \end{pmatrix} + \underset{\sim}{c}_1$$

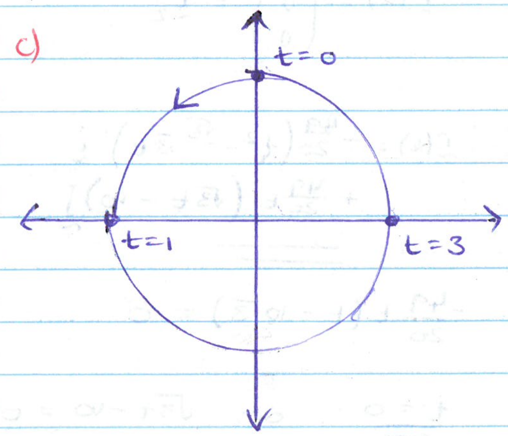
$$\begin{pmatrix} 0 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} + \underset{\sim}{c}_1$$

$$\underset{\sim}{c}_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\therefore \underset{\sim}{r}(t) = \begin{pmatrix} -5\sin\left(\frac{\pi}{2}t\right) \\ 5\cos\left(\frac{\pi}{2}t\right) \end{pmatrix}$$

b) $$\underset{\sim}{r}(3) = \begin{pmatrix} -5\sin\left(\frac{3\pi}{2}\right) \\ 5\cos\left(\frac{3\pi}{2}\right) \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$$

c)



$$\underset{\sim}{r}(1) = \begin{pmatrix} -5 \\ 0 \end{pmatrix}$$

d) $$\int_0^3 \begin{pmatrix} -\frac{5\pi}{2}\cos\left(\frac{\pi}{2}t\right) \\ -\frac{5\pi}{2}\cos\left(\frac{\pi}{2}t\right) \end{pmatrix} dt$$

$$= \left[\begin{matrix} -5\sin\left(\frac{\pi}{2}t\right) \\ 5\cos\left(\frac{\pi}{2}t\right) \end{matrix}\right]_0^3$$

$$= \begin{pmatrix} 5 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 5 \end{pmatrix}$$

$$= \begin{pmatrix} 5 \\ -5 \end{pmatrix}$$

Displacement from $t=0$ directly to $t=3$.

$$\left|\int_0^3 \underset{\sim}{v}(t)\,dt\right| = \left|\begin{pmatrix} 5 \\ -5 \end{pmatrix}\right|$$

$$= \sqrt{25+25}$$

$$= 5\sqrt{2}\text{ m.}$$

Distance from $t=0$ to $t=3$ (direct).

$$\int_0^3 |\underset{\sim}{v}(t)|\,dt$$

$$= \int_0^3 \sqrt{\frac{25\pi^2}{4}\cos^2\left(\frac{\pi}{2}t\right) + \frac{25\pi^2}{4}\sin^2\left(\frac{\pi}{2}t\right)}\,dt$$

$$= \int_0^3 \sqrt{\frac{25\pi^2}{4}(1)}\,dt$$

$$= \int_0^3 \frac{5\pi}{2}\,dt$$

$$= \left[\frac{5\pi}{2}t\right]_0^3$$

$$= \frac{15\pi}{2}\text{ m}$$

Distance around circumference from $t=0$ to $t=3$.

Q8 $$\underset{\sim}{r}(t) = \begin{pmatrix} -2\sin\left(\frac{\pi}{6}t\right) \\ 3\cos\left(\frac{\pi}{6}t\right) \end{pmatrix}$$

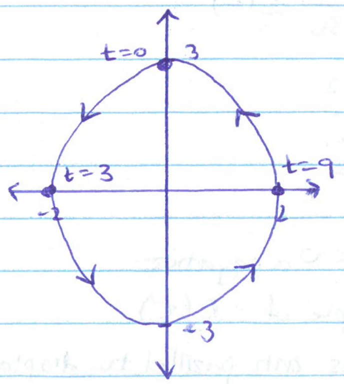
b) $x = -2\sin\left(\frac{\pi}{6}t\right)$ $\qquad y = 3\cos\left(\frac{\pi}{6}t\right)$

$\frac{x}{2} = -\sin\left(\frac{\pi}{6}t\right)$ $\qquad \frac{y}{3} = \cos\left(\frac{\pi}{6}t\right)$

$\frac{x^2}{4} = \sin^2\left(\frac{\pi}{6}t\right)$ $\qquad \frac{y^2}{9} = \cos^2\left(\frac{\pi}{6}t\right)$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

a)



$\underset{\sim}{r}(0) = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$

$\underset{\sim}{r}(3) = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$

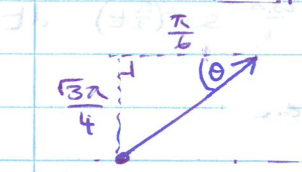
$\underset{\sim}{r}(9) = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

c) $\underset{\sim}{r}(t) = \begin{pmatrix} -2\sin\left(\frac{\pi t}{6}\right) \\ 3\cos\left(\frac{\pi t}{6}\right) \end{pmatrix}$

$\underset{\sim}{v}(t) = \begin{pmatrix} -\frac{\pi}{3}\cos\left(\frac{\pi}{6}t\right) \\ -\frac{\pi}{2}\sin\left(\frac{\pi}{6}t\right) \end{pmatrix}$

$\underset{\sim}{v}(8) = \begin{pmatrix} -\frac{\pi}{3}\cos\left(\frac{4\pi}{3}\right) \\ -\frac{\pi}{2}\sin\left(\frac{4\pi}{3}\right) \end{pmatrix}$

$= \begin{pmatrix} \frac{\pi}{6} \\ \frac{\sqrt{3}\pi}{4} \end{pmatrix}$



$\tan\theta = \frac{\sqrt{3}\pi}{4} \div \frac{\pi}{6}$

$= \frac{6\sqrt{3}}{4}$

$= \frac{3\sqrt{3}}{2}$

$\theta = \tan^{-1}\left(\frac{3\sqrt{3}}{2}\right)$

$\approx 1.20^c$ (2dp)

d) $\underset{\sim}{a}(t) = \begin{pmatrix} \frac{\pi^2}{18}\sin\left(\frac{\pi}{6}t\right) \\ -\frac{\pi^2}{12}\cos\left(\frac{\pi}{6}t\right) \end{pmatrix}$

$= \begin{pmatrix} \frac{2\pi^2}{36}\sin\left(\frac{\pi}{6}t\right) \\ \frac{-3\pi^2}{36}\cos\left(\frac{\pi}{6}t\right) \end{pmatrix}$

$= -\frac{\pi^2}{36}\begin{pmatrix} -2\sin\left(\frac{\pi}{6}t\right) \\ 3\cos\left(\frac{\pi}{6}t\right) \end{pmatrix}$

$= -\frac{\pi^2}{36}\underset{\sim}{r}(t)$

$\therefore k = \frac{\pi^2}{36}$

Given $\underset{\sim}{a}(t)$ is a negative scalar multiple of $\underset{\sim}{r}(t)$ then $\underset{\sim}{a}(t)$ is anti-parallel to displacement

$\Rightarrow$ ✓ inward acceleration

Q9. $\underset{\sim}{a}(t) = \begin{pmatrix} -9.8\sin 30° \\ -9.8\cos 30° \end{pmatrix}$

$= \begin{pmatrix} -4.9 \\ -4.9\sqrt{3} \end{pmatrix}$

$\underset{\sim}{v}(t) = \begin{pmatrix} -4.9t \\ -4.9\sqrt{3}\,t \end{pmatrix} + \underset{\sim}{c}_1$

$\underset{\sim}{v}(0) = \begin{pmatrix} 49\cos 30° \\ 49\sin 30° \end{pmatrix} = \underset{\sim}{c}_1$

$\underset{\sim}{v}(t) = \begin{pmatrix} -4.9t + 49\cos 30° \\ -4.9\sqrt{3}t + 49\sin 30° \end{pmatrix}$

$= \begin{pmatrix} -4.9t + \frac{49\sqrt{3}}{2} \\ -4.9\sqrt{3}t + \frac{49}{2} \end{pmatrix}$

$\underset{\sim}{r}(t) = \begin{pmatrix} -\frac{4.9t^2}{2} + \frac{49\sqrt{3}}{2}t \\ -\frac{4.9\sqrt{3}}{2}t^2 + \frac{49}{2}t \end{pmatrix} + \underset{\sim}{c}_2$

$\underset{\sim}{r}(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \underset{\sim}{c}_2$

$\therefore \underset{\sim}{r}(t) = -\frac{49}{20}t\left(t - 10\sqrt{3}\right)\underset{\sim}{i} - \frac{49}{20}t\left(\sqrt{3}t - 10\right)\underset{\sim}{j}$.

$-\frac{49}{20}t\left(\sqrt{3}t - 10\right) = 0$

$t = 0$ (ignore) or $\sqrt{3}t - 10 = 0$

$t = \frac{10}{\sqrt{3}}$

$= \frac{10\sqrt{3}}{3}$ secs

Q10.

$\underset{\sim}{v}(t) = \begin{pmatrix} 1 - \cos t \\ \sin t \end{pmatrix}$

a) $\underset{\sim}{r}(t) = \begin{pmatrix} t - \sin t \\ -\cos t \end{pmatrix} + \underset{\sim}{c}_1$

$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} + \underset{\sim}{c}_1$

$\underset{\sim}{c}_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\therefore \underset{\sim}{r}(t) = \begin{pmatrix} t - \sin t \\ 1 - \cos t \end{pmatrix}$ m

b) Period is 2π.

$\underset{\sim}{r}(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\underset{\sim}{r}(\pi) = \begin{pmatrix} \pi - 0 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} \pi \\ 2 \end{pmatrix}$

2

$\therefore$ Diameter is 2m.

0

c)

i) $\underset{\sim}{r}(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\underset{\sim}{v}(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

ii) $\underset{\sim}{r}\left(\frac{\pi}{2}\right) = \begin{pmatrix} \frac{\pi}{2} - 1 \\ 1 \end{pmatrix}$

$\underset{\sim}{v}\left(\frac{\pi}{2}\right) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

iii) $\underset{\sim}{r}(\pi) = \begin{pmatrix} \pi \\ 2 \end{pmatrix}$

$\underset{\sim}{v}(\pi) = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

iv) $\underset{\sim}{r}\left(\frac{3\pi}{2}\right) = \begin{pmatrix} \frac{3\pi}{2} + 1 \\ 0 \end{pmatrix}$

$\underset{\sim}{v}\left(\frac{3\pi}{2}\right) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

d) Graph parametrically

$x = t - \sin t$

$y = 1 - \cos t$